Strategies for PROBLEN SOLIVNE:
Equip Kids to Solve Math Problems with Confidence


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# Equip Kids to Solve Math Problems With Confidence 

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## Strategies for Problem Solving:

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## Table of Contents:

Introduction ..... 5
Chapter 1: The Problem Solving Process ..... 7
Chapter 2: Teach Kids to Problem Solve by Drawing a Picture ..... 11
Chapter 3: Teach Kids to Problem Solve by Solving an Easier Problem. ..... 13
Chapter 4: Teach Kids to Problem Solve by Working Backwards. ..... 15
Chapter 5: Teach Kids to Problem Solve by Making a List. ..... 19
Chapter 6: Teach Kids to Problem Solve by Finding a Pattern ..... 22
Chapter 7: Teach Kids to Problem Solve by Guess and Check ..... 27
Chapter 8: Final Tips for Math Success ..... 32
Printable Summary Page ..... 34
References ..... 35

## Introduction:

There is one phrase that tends to evoke dread and anxiety in the hearts of countless students: "Solve the following word problems." Word problems often seem like nonsense and infinitely more confusing and challenging than a simple and straightforward math problem. Why is that? I think for many students, word problems are overwhelming. There are so many words and things to think about and consider, it can be hard to know where to start. And often, word problems include unnecessary information, making it a challenge to wade through everything and find what you actually need to solve it. This does not even consider students who struggle to read, or English language learners, who are challenged just by trying to comprehend the words, much less the math involved.

So what is the solution? Do not ever assign word (or story) problems? Of course not! It is important for students to be exposed to real life applications, because that is the whole point of learning math. In addition,
it is never a bad thing for students to be challenged. That is how we learn new things!

Instead, work to equip students and give them tools to use to increase their confidence. Help them wade through word problems so that they do not seem as overwhelming and daunting. Teach them to be problem solvers.

I hope that this short book will give you a great starting point, and will aid you and your students as you seek to solve problems together.

# The Problem Solving Process: 

Whenever students are faced with a word problem, there are some helpful steps to take and things to think about before they ever actually start solving. This process is useful no matter the problem, and can help students take a step back and think through the problem. Hopefully in so doing, they will not feel overwhelmed by the task before them.

## Step 1: Understanding the Question

This is the step that I spend the most time on with students. It is the most essential part of the problem solving process, because if you do not actually know what the question is asking or what you are looking for, how will you ever figure out how to find the solution?

In order to understand the question, have students first read through the problem once. This will give them an idea of the context and allow them to see all the pieces together. Then have them go back through more slowly and circle or highlight important information.

They can then use what they have read and circled to make sure they understand the question correctly. To do this they should ask two questions: "What do I know?" and, "What do I need to know?"

In trying to understand the question, it is also helpful for students (especially for those who struggle) to stop and rewrite or restate the question in their own words. If a student cannot explain what the question is asking in their own words, they should not move on. Again, you cannot find the solution if you do not know the problem.

Do not be afraid to spend a significant amount of time focusing on understanding the question. As students get older and the math gets more complicated, the problems will become more involved. Often, solving a word problem will require multiple steps, and if students have not determined what the question is asking, they may find a solution that is correct (in terms of the math), but does not actually answer the question.

After sufficiently understanding what is given and what they are looking for, students can move on to step 2.

## Step 2: Make a Plan

Once students understand the question and have determined all the important information given, they can come up with a plan to solve it. They should ask themselves, "What is the question asking me to find?" and determine which problem solving strategy is going to be the most beneficial. For more complicated problems, it may be helpful to quickly write a list of the information needed in order to answer the question. It may be that they need to solve for some other necessary piece of
information first in order to find the solution to the question. Making a list will help them stay on track and not lose sight of the final destination in the problem solving process.

## Step 3: Solve the Problem

Once they understand the question and have a plan in place, they simply follow the plan. This is the part where they actually do the math.

## Step 4: Check the Solution

After working through the plan and coming up with a solution, it is important to see first of all if the solution makes sense. Then, if it seems to be reasonable, check to be sure that it is accurate. In other words, do a quick estimate first, and then check to be sure the answer is exact.

If the answer is not reasonable (i.e. it is negative but should be positive or is very large when it should be small), the student should go back to their calculations and see if there is a simple computational error. If they cannot find an error, they should go back to the problem-solving plan and determine a different method to solve, perhaps using a different strategy.

While this four-step process sounds long and tedious, most of this is done in your head as you read through the problem. As you go through and highlight important information, your brain is already forming a strategy to attack the problem. As you read the question, you may think about it in your own words and come up with a logical plan. Sometimes, the steps may overlap as you make sense of the problem. The more problems you solve, the more this process becomes a habit that does not
need to be spelled out one step at a time. The goal is that this process would be so familiar to your students that it is second nature.

Now that you have a general idea of how to think through any word problem, here are some strategies to use for various types of problems.

## Problem Solve by Drawing Q Picture:

I am a very visual learner. Whenever I am facing a word problem of any kind, my initial reaction is to draw a picture. Even if it is a fairly simple problem and I think I already know how to solve it (or even already know the answer), I will almost always still draw a picture. While this is an especially useful strategy for visual learners, I believe that problem solving by drawing a picture can be helpful for any student.

Maybe I am drawn to this strategy because I am such a great artist. No, that is definitely not it! I believe it is because seeing a visual representation of the problem can put things in perspective, help organize the information, and enable students to make connections that may not have been seen otherwise.

So while I know that not everyone is a visual learner, I believe this is still an important and helpful problem solving strategy. Especially if you are stuck and do not know where to go or what to do. Then you have nothing to lose, right?

So when is this strategy useful? Anytime the problem presents a situation that can be represented with a picture or diagram.

When I was teaching high school, I would often encourage students to draw a picture when working on distance/rate/time problems. It is very easy to get bogged down in all the details and numbers, especially if the problem includes unnecessary information (details that you do not really need to know in order to solve it). Wading through everything you are given and making sense of what is important is often easier when you draw a picture.

It is also incredibly important to draw a picture when working on geometry or right triangle trigonometry problems. Even if you know how to solve it without a picture, you will greatly increase you chances of a careless mistake if you do not take a few extra seconds to sketch a picture.

One important thing to remember, however, is that the picture does not need to be pretty. In fact, in many cases it may not even be a picture, just a visual representation of the information. And that is ok! The point is to help you solve the math problem, not to win an art award.

# Problem Solve by Solving an Easier Problem: 

Hungarian Mathematician, George Polya, put it this way in his small but important work, How to Solve It (1965):
"If you can't solve a problem, then there is an easier problem you can solve: find it."

If a problem seems overwhelming, has a lot of steps, or very large numbers, try to solve a simpler version or a similar problem to help get you going in the right direction.

For example, I was recently working with a third grader on some story problems. The question said,
"Brielle and Lindsay found 527 bugs on Wednesday. On Thursday, they discovered 374 bugs. How many more bugs did the girls find on Wednesday than Thursday?"

My student immediately wrote 527 over 374 on her paper and drew an equals sign. So I asked, "Ok, we have these two numbers, what do we need to do with them?" She did not understand the question well enough to know if she needed to add or subtract, so we looked at a simpler problem.

I drew a chart with Wednesday and Thursday and listed how many bugs they found on each day. First, we talked about what the question was asking (how many more, meaning how much bigger or what is the difference between the two).

Then, I simplified things by changing the numbers from 527 and 374 to simply 5 and 3 . We then plotted the numbers on a number line and saw that 5 is 2 more than 3 . So I asked, "What operation can we use to find that 5 is 2 more than 3 ?" She answered with "Subtraction," and I was able to show her that if subtraction worked in this simple version, it would work with the larger numbers as well.

Taking the time to think through this problem was important because she struggles with knowing what to do with all the pieces of information given in word problems. If I had simply let her guess and work it out (using addition), and then said, "Oh, actually you're supposed to subtract here. Try again!" she likely would have made the same mistake the next time.

Similarly, students are often thrown off by variables in Algebra. Whenever a student gets stuck or confused, I replace the variable with a number (because a variable represents a number, it is simply not known which one). By simplifying the problem, I can help students see what operation to use or steps to take to solve it.

# Problem Solve by Working Backuras: 

Before students can learn to recognize when this is a helpful strategy, they must understand what it means. Working backwards is to start with the final solution and work back one step at a time to get to the beginning.

It may also be helpful for students to understand that this is useful in many aspects of life, not just solving math problems. For instance, when I plan a math lesson, I think about what I want students to know and be able to do, and then work backwards from there. Or when I have a set of goals that I want to achieve, I work backwards from the goal to come up with steps that will (hopefully) get me there. And as Stephen Covey says in his book, 7 Habits of Highly Effective People, effective people "start with the end in mind."

To help show your students what this looks like, you might start by thinking about directions. Write out some basic directions from home to school:

Start: Home<br>Turn right on Gray St.<br>Turn left on Sycamore Ln.<br>Turn left on Rose Dr.<br>Turn right on Schoolhouse Rd.<br>End: School

Ask students to then use this information to give directions from the school back to home. Depending on the age of your students, you may even want to draw a map so they can see clearly that they have to do the opposite as they make their way back home from school. In other words, they need to "undo" each turn to get back, i.e. turn left on Schoolhouse Rd. and then right on Rose Dr. etc.

In math, these are called inverse operations. When using the "work backwards" strategy, each operation must be reversed to get back to the beginning. So if working forwards requires addition, when students work backwards they will need to subtract. And if they multiply working forwards, they must divide when working backwards.

Once students understand inverse operations, and know that they must start with the solution and work back to the beginning, they will need to learn to recognize the types of problems that require working backwards.

In general, working backwards can be used to solve problems that list a series of events or a sequence of steps.

Here's an example:

Sam's mom left a plate of cookies on the counter. Sam ate 2 of them, his dad ate 3 of them, and they gave 12 to the neighbor. At the end of the day, only 4 cookies were left on the plate. How many cookies did she make altogether?

In this case, we know that the final cookie amount is 4 . So if we work backwards to "put back" all the cookies that were taken or eaten, we can figure out what number they started with.

The cookies being taken away denotes subtraction. Thus, to get back to the original number we have to do the opposite: add. If you take the 4 that are left and add the 12 given to the neighbors, and add the 3 that Dad ate and add the 2 that Sam ate, we find that Sam's mom made 21 cookies.

You may want to give students a few similar problems to let them see when working backwards is useful, and what problems look like that require working backwards to solve.

This is also especially useful when working on multiple-choice math problems. Though this is not my favorite, and I do not want to train kids to "work the test," there will be times when they are taking standardized tests or the SAT/ACT, so it is important to know how to work backwards.

When given multiple-choice answers, more often than not it is possible to use the answer choices to work backwards by plugging the choices back in to find the true solution. The answer choice that makes the problem true is the correct choice.

This is a different kind of "working backwards," but it is important to cover this with your students nonetheless (especially when they are preparing for a state test or SAT/ACT).

# Problert Solve by Mreizing  

This was always a hard approach for me personally because it does not seem like math, and can often be time consuming. I mean who really wants to sit and list out all the possible solutions to a math problem? But it is a very useful strategy, and as we will see, learning to be organized and systematic is the key.

So when is it useful to make a list? Any time you have a problem that has more than one solution, or you are trying to solve a combination problem, it is helpful to make a list.

But not just any list of possibilities. That will feel useless and frustrating if you are just trying to pull out possibilities from anywhere. It is very likely possibilities will be skipped or repeated that way, making the final solution wrong.

In addition, it will be more time consuming to make a list if you do not have a systematic approach to it, which is probably why I was never a fan as a kid. No one wants to just sit and stare at the paper hoping solutions will pop into their brain.

Organizing the information keeps you on track and ensures that all the possible solutions will be found. There are different ways to organize information, but the idea is to exhaust all the possibilities with one part of your list before moving on.

For example, say you are trying to figure out all the different combinations of ice cream toppings at your local ice cream shop. They have 3 different flavors (chocolate, vanilla and strawberry) but also have 4 different toppings (nuts, whipped cream, chocolate candies and gummy bears).

If you just start listing different possibilities without any kind of structure, you are bound to get lost in your list and miss something. Instead, list all the possibilities for chocolate ice cream before moving on to vanilla:

Chocolate: just chocolate (no toppings), chocolate with nuts, chocolate with whipped cream, chocolate with candies and chocolate with gummy bears.

Now we see that there are 5 possibilities if you get chocolate ice cream, and so we can move on to vanilla, and then strawberry. Some students may even notice that there will be 5 possibilities for each flavor, and thus multiply $3 \times 5$ without completing the list. (That's another great strategy: look for patterns).

Even if a pattern is not discovered, however, completing the list in an organized, systematic way will ensure all possibilities are covered and the final solution will be 15 possible ice cream flavor combinations.

Another way to organize the list is to make a tree diagram. Here is another example:

Sarah is on vacation and brought 3 pairs of pants (blue, black, white) and 3 shirts (pink, yellow and green). How many different outfit combinations can she make?

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It is now clear that there are 9 possible outfit combinations.

Using a tree diagram is a great way to keep the information organized, especially if you have kids who struggle with keeping track of their list.

# Trocter Solve oy Trating e Tais anc eincins e Paterno 

One important math concept that children begin to learn and apply in elementary school is reading and using a table. This is essential knowledge, because we encounter tables of data all the time in our everyday lives. But it is not just important that kids can read and answer questions based on information in a table. It is also important that they know how to create their own table and then use it to solve problems, find patterns, graph equations, and so on. And while some may think of these as two different things, I think problem solving by making a table and looking for a pattern go hand in hand.

So when should kids try to solve a problem by making a table? Well, when the problem gives a set of data, or a pattern that is continuing and can be arranged in a table.

As I mentioned when I discussed problem solving by making a list, finding a pattern can be immensely helpful and save a lot of time when working on a word problem. Sometimes, however, a student may not recognize the pattern right away, or may get bogged down with all the details of the question.

Setting up a table and filling in the information given in the question is a great way to organize things and provide a visual so that the "rule" of the pattern can be determined. The "rule" can then be used to find the answer to the question. This removes the tedious work of completing a table, which is especially nice if a lot of computation is involved.

But a table is also great for kids who struggle with math, because it gives them a way to get to the solution even if they have a hard time finding the pattern, or are not confident that they are using the "rule" correctly. Even though using a known pattern can save you time, and eliminate the need to fill out the entire table, it is not necessary. A student who is unsure could simply continue filling out their table until they reach the solution they are looking for.

Helping students learn how to set up a table is also helpful because they can use it to organize information (much like making a list) even if there is not a pattern to be found. Completing the table in a systematic way ensures that nothing is left out.

If your students are just learning how to read and create tables, I would suggest having them circle their answer in the table to show that they understood the question and knew where in the table to find the answer.

If you have older students, encourage them to find a pattern in the table and explain it in both words as well as mathematical symbols and/or an equation. This will help them form connections and increase number sense. It will also help them see how to use their "rule" or equation to solve the given question as well as make predictions about the data.

It is also important for students to consider whether or not their pattern will continue predictably. In some instances, the pattern may look one way for the first few entries, then change. This is important to consider as problems get more challenging.

There are tons of examples of problems where creating a table and finding a pattern is a useful strategy, but here is just one example for you:

Ben decides to prepare for a marathon by running 10 minutes a day, six days a week. Each week, he increases his time running by 2 minutes. How many minutes will he run in week 8?

Included in the table is the week number (we are looking at weeks 18), as well as the number of minutes per day and the total minutes for the week. The first step is to fill in the first couple of weeks by calculating the total time.

Once you have found the total minutes for weeks 1-3, you may see the pattern and be able to calculate the total minutes for week 8.

| Week Number | Total Minutes Per Day | Total Minutes for the <br> Week |
| :---: | :---: | :---: |
| 1 | 10 | 60 |
| 2 | 12 | 72 |
| 3 | 14 | 84 |

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If not, however, simply continue with the table until you get to week 8 , and then you will have your answer (144 minutes).

| Week Number | Total Minutes Per Day | Total Minutes for the <br> Week |
| :---: | :---: | :---: |
| 1 | 10 | 60 |
| 2 | 12 | 72 |
| 3 | 14 | 84 |
| 4 | 16 | 96 |
| 5 | 18 | 108 |
| 7 | 20 | 120 |
| 8 | 22 | 132 |
|  | 24 |  |
| 7 |  | 144 |

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I think it is especially important to make it clear to students that it is perfectly acceptable to complete the entire table (or continue a given table) if they do not see or do not know how to use the pattern rule to solve the problem.

I was working with a student once who was given a table, but then asked a question about information not included in that table. She was able to tell me the pattern she saw, but was not able to correctly use the "rule" to find the answer.

I encouraged her to simply extend the table until she found what she needed. Then I showed her how to use the "rule" of the pattern to get the same answer.

# Problem Solve by Guess and Check: 

As with the other strategies I have discussed, it is important to help kids understand how to use this method so that they are not randomly pulling answers out of their head and wasting time.

You may hear the name of this strategy and think, "Guess? Isn't the whole point of math instruction to teach kids to solve problems so that they are no longer merely guessing?"

While it is certainly true that we do not want kids to simply guess random answers for every math problem they ever encounter, there are certain instances when educated guesses are important, valid and useful.

For instance, learning and understanding how to accurately estimate is an important mathematical skill. A good estimate, however, is not just a random guess. It takes effort and logic to formulate an estimate that
makes sense and is (hopefully) close to the correct answer. Similarly, solving problems using guess and check is a process that requires logic and an understanding of the question so that it can be done in a way that is organized and time saving.

So what does guess and check mean? To be more specific, this strategy should be called, "Guess, Check and Revise." The basic structure of the strategy is to form an educated guess, check your solution to see if it works and solves the problem, and if not, revise your guess based on whether it is too high or too low, etc.

This is a useful strategy when you are given the total and you are asked to find the kinds or number of things making up the total. It is also beneficial when the question asks for the value of two or more different kinds of things.

For instance, you might be asked how many girls and how many boys are in the class, or how many cats and how many dogs does a pet owner possess.

When guess and check seems like an appropriate strategy, it will be helpful and necessary to then organize the information in a table or list to keep track of the different guesses. This provides a visual of the important information, and also helps to ensure that subsequent guesses are logical and not random.

To begin, students should make a guess using what they know from the problem. This first guess can be anything at all, so long as it follows the criteria given. Then, once a guess is made, students can begin to make
more educated guesses based on how close they are to the correct answer.

For example, if their initial guess gives a total that is too high, they need to choose smaller numbers for their next guess. Likewise, if their guess gives a total that is too low, they need to choose larger numbers.

The most important thing for students to understand when using this method is that after their initial guess, they should work towards getting closer to the correct answer by making logical changes to their guess. They should not be choosing random numbers anymore.

Here is an example to consider:

In Ms. Brown's class, there are 24 students. There are 6 more girls than boys. How many boys and girls are there?

Because you know the class total (24), and are asked to find more than one value (number of boys and number of girls), it can be solved using the guess and check method.

To organize the question, form a table with boys, girls and the class total. Because you know there are 6 more girls than boys, we can guess a number for the boys, and then calculate the girls and the total from there.

An initial guess of 12 boys means that there would be 18 girls, giving a total class size of 30 . The total, however, should only be 24 , which means the guess was too high. Knowing this, the number of boys should be revised and the total recalculated.

| Boys | Girls $(\mathbf{+ 6 )}$ | Total Students |
| :---: | :---: | :---: |
| 12 | 18 | 30 |

Lowering the number of boys to 10 would mean there are 16 girls, which gives a class total of 26 .

| Boys | Girls ( $\mathbf{+ 6}$ ) | Total Students |
| :---: | :---: | :---: |
| 12 | 18 | 30 |
| 10 | 16 | 26 |

This is still just a little bit too high, so once again, revise the guess to 9 boys. If there are 9 boys, that would mean there are 15 girls, which gives a class total of 24 .

| Boys | Girls (+6) | Total Students |
| :---: | :---: | :---: |
| 12 | 18 | 30 |
| 10 | 16 | 26 |
| 9 | 15 | 24 |

Therefore, the solution is 9 boys and 15 girls.

This is a fairly simple example, and likely you will have students who can solve this problem without writing out a table and forming multiple guesses. For students who struggle with math, however, this problem may seem overwhelming and complicated. By giving them a starting point and helping them learn to make more educated guesses, you can equip them to not only solve word problems, but to feel more confident in tackling them.

This is also a good strategy because it helps kids see that it is ok to make mistakes and that we should not expect to get the right answer on the first try, but rather, we should expect to make mistakes. The key is to know how to use our mistake to learn and find the right answer.

## Tina Tips for Math Success:

Hopefully you now feel prepared to teach these strategies to your students or children as they learn to be thinkers and problem solvers. While it is my desire that kids learn to enjoy math and not feel overwhelmed or anxious at the thought of a word problem, one of the things that will be most beneficial in the learning process is time. Time to practice, time to learn from mistakes, and time to become better at computations.

And although I firmly believe that math is a learned skill that all children are capable of, it is not something that can be learned and mastered overnight. Encourage kids to persevere, and have patience in their math endeavors.

You may also want to encourage and remind students that they may often use multiple strategies in the same problem. This is a great practice and will help them better make sense of problems in general.

## Other Helpful Strategies for Math Word Problems:

- Write an equation (or number sentence): Some students may prefer writing out a number sentence with the given information to drawing a picture. This is also a great way to evaluate, make sense of, and solve the problem.
- Use a number line or hundreds chart: I love using a number line to visualize math problems. Help students learn to sketch a logical piece of the number line to aid in solving the problem. Make sure they know that it can include positive or negative numbers, and that there are an infinite number of numbers in between each number.
- Skip it and return later: Okay, so this is not a strategy, per se, but sometimes the best solution is to take a break. Especially if frustration or anxiety is setting in. Perhaps working on a different problem will spark an idea for the previous question. And sometimes it is simply helpful to step away and breathe for a bit.

Please use the tips and information in this ebook as a reference and guide. There is always more than one way to solve a math problem and these are simply meant to be starting points for students, not the final or comprehensive list of strategies.

The goal in teaching mathematics is to produce independent thinkers and problem solvers. So if they approach a problem in their own way, embrace it (as long as it is mathematically sound). And be encouraged that they have come up with a brand new problem solving strategy!

## Problem Solving strategies Guide

| Strategy | When to Use It | Example |
| :---: | :---: | :---: |
| Draw a Picture | The problem presents a situation that can be represented with a picture or diagram. | There are 3 birds in the nest and 12 flying. How many birds are there altogether? |
| Solve an Easier Problem | The problem uses large numbers or has a lot of variables. | The baker made 624 cupcakes. There were only 156 left at the end of the day. How many cupcakes sold? |
| Work Backwards | The problem lists a series of events or a sequence of steps. (This is also helpful with multiple-choice problems) | At 8 pm. 6 people left the party. At 9 pm. 8 people left the party. At 10 pm. 2 people left and 4 remained. How many people were at the party altogether? |
| Make a Lis $\dagger$ | The problem has more than one solution or is a combination problem. | Todd brings a blue. yellow and white shirt on vacation. He also brings blue pants and khaki pants. How many different outfits can he make? |
| Make a Table/Find a Pattern | The problem gives a set of data or a continuing pattern that can be arranged in a table. | Sam paid a $\$ 50$ deposit to rent a truck and then pays $\$ 12$ per day. How much will it cost if he rents it for 6 days? |
| Guess and Check | The problem gives the total and you are looking for two or more parts of the total or multiple kinds of things. | At Christmas. the grocery sells ornaments. There are 8 more red than gold. and 6 less green than red. If there are 40 altogether. how many does the store have of each color? |
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